

Integration by Parts

by Jim Johnston

$$\int u dv = uv - \int v du$$

Example

(a) Find the integral of $\int (x - 9) \ln x dx$

Let $u = x - 9, du = dx$

Let $dv = \ln x dx, v = \int \ln x dx$

$$\int (x - 9) \ln x dx = x(x - 9)(\ln x - 1) - \iint \ln x dx$$

(b) Find the integral of $\int \ln x dx$

Let $u = \ln x, du = \frac{1}{x} dx$

Let $dv = dx, v = \int dx = x$

$$\int \ln x dx = x \ln x - \int dx$$

$$\int \ln x dx = x \ln x - x = x(\ln x - 1)$$

(c) Continuing back to example (a) we now have,

$$\int (x - 9) \ln x dx = x(x - 9)(\ln x - 1) - \int x(\ln x - 1) dx$$

(d) Find the integral of $\int x(\ln x - 1) dx$

Let $u = \ln x - 1, du = \frac{1}{x} dx$

Let $dv = x dx, v = \int x dx = \frac{x^2}{2}$

$$\int x(\ln x - 1) dx = (\ln x - 1) \left(\frac{x^2}{2} \right) - \int \frac{x}{2} dx$$

$$\int x(\ln x - 1) dx = (\ln x - 1) \left(\frac{x^2}{2} \right) - \frac{x^2}{4}$$

(e) Continuing back to example (a) we now have,

$$\int (x - 9) \ln x \, dx = x(x - 9)(\ln x - 1) - \left[(\ln x - 1) \left(\frac{x^2}{2} \right) - \frac{x^2}{4} \right]$$
$$\int (x - 9) \ln x \, dx = x(x - 9)(\ln x - 1) - (\ln x - 1) \left(\frac{x^2}{2} \right) + \frac{x^2}{4}$$